State-dependent local projections*

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Abstract

Do state-dependent local projections asymptotically recover the population responses of macroeconomic aggregates to structural shocks? The answer to this question depends on how the state of the economy is determined and on the magnitude of the shocks. When the state is exogenous, the local projection estimator recovers the population response regardless of the shock size. When the state depends on macroeconomic shocks, as is common in empirical work, local projections only recover the conditional response to an infinitesimal shock, but not the responses to larger shocks of interest in many applications. Simulations suggest that impulse responses may be off by as much as 82 percent and fiscal multipliers by as much as 40 percent.

JEL codes : C22, C32, H20, C51, E32, E52, E60, E62

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1 Introduction

The recent empirical macroeconomics literature has emphasized the importance of allowing for nonlinearities when estimating the effects of exogenous shocks on macroeconomic variables of interest. A key question in empirical work is how impulse response functions depend on the state of the economy. For example, many studies estimating the government spending multiplier allow for the possibility that this multiplier may be different during recessions and expansions (e.g., Auerbach and Gorodnichenko (2012, 2013a,b), Bachmann and Sims (2012), Owyang, Ramey and Zubairy (2013), Caggiano, Castelnuovo, Colombo and Nodari (2015), Ramey and Zubairy (2018), Alloza (2022), and Ghassibe and Zanetti (2020)). There is also a related literature on the dependence of tax multipliers on the business cycle (e.g., Candelon and Lieb (2013), Alesina, Azzalini, Favero, Giavazzi and Miano (2018), Sims and Wolff (2018), Eskandari (2019), and Demirel (2021)). Similar questions arise in many other contexts including the analysis of monetary policy shocks. For example, Santoro, Petrella, Pfajfar and Gaffeo (2014), Tenreyro and Thwaites (2016), Angrist, Jordà and Kuersteiner (2018), Barnichon and Matthes (2018) and Klepacz (2020) allow the responses to monetary policy shocks to vary as a function of the state of the economy. Yet another example of the estimation of state-dependent responses is the work of Caggiano, Castelnuovo and Groshenny (2014) who examine the dependence of the effects of uncertainty shocks on whether the economy is in recession or expansion.

Many of these studies rely on a variant of the local projection (LP) approach of Jordà (2005, 2009) (see also Dufour and Renault (1998) and Chang and Sakata (2007)) to estimate the state-dependent impulse response functions. For example, given an observed policy shock series ε_{1t}, a state-dependent local projection estimates the dynamic effect of ε_{1t} on the scalar variable y_{t+h} conditionally on the state of the economy by a set of regressions for each horizon h:

$$y_{t+h} = S_{t-1}\left[b_h\left(1\right)\varepsilon_{1t} + \pi'_{E,h}z_{t-1}\right] + \left(1 - S_{t-1}\right)\left[b_h\left(0\right)\varepsilon_{1t} + \pi'_{R,h}z_{t-1}\right] + v_{t+h}, \tag{1}$$

where S_{t-1} is a variable that takes the value 1 if the economy is in expansion (E) in period $t-1$ and 0 if it is in recession (R). The least squares estimate $\hat{b}_h\left(1\right)$ of the slope coefficient associated with $\varepsilon_{1t}S_{t-1}$ is usually interpreted as the impulse response of y_{t+h}, conditionally on $S_{t-1} = 1$, while $\hat{b}_h\left(0\right)$ is interpreted as the response of y_{t+h} when conditioning on $S_{t-1} = 0$. The regressor z_{t-1} includes lags of all model variables.

One argument for using state-dependent local projections rather than state-dependent structural vector autoregressive (VAR) models has been their computational simplicity. Estimating impulse responses in state-dependent VAR models by numerical methods tends to be computationally more challenging than the estimation of state-dependent local projections by the method of least squares. A related argument has been that LP estimators dispense with the need to estimate equations for dependent variables other than the outcome variable of interest. Finally, unlike state-dependent VAR models, state-dependent local projections may be estimated without having to specify the process governing the transition from one state to the other. As a result, Ramey's (2016) handbook chapter concludes that "if one is interested in estimating state dependent models, the [...] local projection method is a simple way to estimate such a model and calculate impulse response functions (p. 87)".

Table 1 lists 45 journal articles published over the last ten years in general interest journals and field journals in macroeconomics, public economics, international economics, and applied econometrics that use this approach. State-dependent LP estimators are also discussed at length in book chapters (e.g., Auerbach and Gorodnichenko (2013b), Auerbach and Gorodnichenko (2017), Ramey (2016)) and they continue to be used extensively in recent working papers (e.g., Ahir, Bloom and Furceri (2022), Alloza, Gonzalo and Sanz (2021), Cloyne, Jordà and Taylor (2023), De Ridder, Hannon and Pfajfar (2020), Eskandari (2019), Ferriere and Navarro (2020), Jo and Zubairy (2022), Klepacz (2021)).

Perhaps surprisingly, despite its widespread application, the validity of the LP approach to estimating state-dependent impulse responses has not been established to date. It has been taken as self-evident in applied work that the state-dependent LP estimator will be consistent. In this paper, we analyze the validity of the state-dependent LP estimator within a potential outcomes framework. The interpretation of linear local projections within this framework is discussed in Rambachan and Shephard (2021). Related work also includes Cloyne, Jorda and Taylor (2023).

We clarify the conditions under which the state-dependent LP estimator can be expected to recover the population impulse responses in this setting.[1] This task is complicated by the fact that for state-dependent processes there are alternative definitions of the population impulse response one may have in mind. For example, one possible definition of the population response is the average response of the outcome variable to a shock of magnitude δ, conditional on the state of the economy at the time when this shock occurs, building on a large literature

[1]The original LP estimator, as discussed in Jordà (2005, 2009) and Plagborg-Møller and Wolf (2021), did not allow for the impulse response function to change depending on the state of the economy. In this paper we are not concerned with linear approximations to nonlinear processes as in Plagborg-Møller and Wolf (2021), but with approximations that are explicitly state dependent and hence nonlinear.

on nonlinear impulse response analysis in time series econometrics (see, e.g., Koop, Pesaran and Potter (1996), Potter (2000), Gonçalves, Herrera, Kilian and Pesavento (2022)). Another possible definition is the marginal response of the outcome variable to an infinitesimal shock, conditional on the state of the economy at the time of this shock.

We formally show that, depending on how the state of the economy is determined, the state-dependent estimator may be able to recover the population response under one impulse response definition, but not under the other. This result not only affects the interpretation of the state-dependent LP estimator in applied work, but also the conditions for establishing its asymptotic validity. Hence, users need to be explicit about which population response they are interested in recovering. To the extent that this point has been discussed at all, the presumption appears to be that the state-dependent LP estimator in the limit will recover the same conditional response function as a state-dependent structural VAR estimator, provided the VAR model is correctly specified (e.g., Auerbach and Gorodnichenko 2013a, p. 67). Our analysis shows that this asymptotic equivalence does not hold in general.

We find that the validity of the state-dependent estimator and its interpretation depends on whether the state of the economy evolves exogenously with respect to the economy or responds endogenously to macroeconomic shocks. In the former case, the two conditional impulse response definitions above yield the same answer (up to scale). Given that the business cycle is typically defined in terms of outcome variables such as real output or unemployment that endogenously respond to all shocks in the economy, however, this result is of limited applicability in macroeconomics. We show that, when the state of the economy is endogenous with respect to macroeconomic variables, as is typically the case in applied work, much depends on the magnitude δ of the structural shock of interest. When δ is not arbitrarily close to zero, the state-dependent LP estimator will not recover the conditional average response function. However, the state-dependent LP estimator under suitable additional assumptions will recover the conditional marginal response function. Thus, the definition of the population response of interest matters.

This raises the question of how comfortable applied users are with the assumption of infinitesimal δ underlying the conditional marginal responses. The answer is likely to depend on the economic context. For example, Ramey and Zubairy (2018) stress the importance of focusing on large spending shocks in their analysis. When the state of the economy is endogenous, the conditional marginal response function does not capture the responses to such large shocks. These responses can only be estimated by the conditional average response, which the state-dependent LP estimator fails to recover.

The practical problem is that it is hard to tell when δ is small enough to focus on the conditional marginal response. For example, is a 25 basis points shock small? Intuitively, δ is small when the conditional average response nearly coincides with the conditional marginal response. How different these responses are crucially depends on the ratio δ/σ_1, where σ_1 denotes the standard deviation of the shock series. Thus, even $\delta = 0.25$ may be large if σ_1 is sufficiently small. Indeed, such a shock would be only slightly smaller than one standard deviation of typical monthly measures of exogenous monetary policy shocks. Likewise, large ratios are common in studies of fiscal policy shocks. For example, it can be shown that the shock to military spending postulated in Ramey and Zubairy (2018) amounts to a 17 standard deviation shock.

We present simulation evidence that the state-dependent LP impulse response estimator is strongly biased in population when δ is large relative to the standard deviation of the shock, calling into question the use of response estimates and fiscal multipliers based on the assumption of infinitesimal δ. Likewise, fiscal multipliers in realistic settings may easily differ by as much as 40 percent from the population multiplier. We conclude that in many empirically plausible settings, state-dependent LP estimators of impulse responses are not likely to provide economically meaningful estimates. Our analysis, however, suggests an alternative nonparametric estimator of state-dependent average responses that remains valid even when δ is far from zero, regardless of whether the state of the economy is endogenous or exogenous.

The remainder of the paper is organized as follows. In Section 2, we consider a bivariate parametric model for expository purposes. This model is chosen to make the analysis as transparent as possible and to facilitate the derivation of analytical results. This section also defines the population impulse responses of interest. In Section 3, we formally derive the limit of the state-dependent LP estimator under exogenous and under endogenous states. We show that the state-dependent LP estimator is valid when the state of the economy is exogenous regardless of the magnitude δ of the structural shock.[2] We furthermore show that this estimator remains valid under suitable assumptions when the state of the economy is endogenous and δ is infinitesimal. However, it is not valid in general when the state of the economy is endogenous and δ is nonnegligible.

[2]As shown in online Appendix B, this conclusion applies not only to the bivariate example in Section 2, but extends to multivariate models whether the forcing variable is i.i.d., as in our analysis in this paper, or a serially correlated exogenous process, as discussed in Alloza, Gonzalo, and Sanz (2021), or merely predetermined and endogenous.

In Section 4, we quantify by simulation the asymptotic bias of the state-dependent LP estimator of the impulse responses when δ is not close to zero. In Section 5, we examine a representative empirical model of macroeconomic responses to fiscal policy shocks and quantify by simulation the implied asymptotic bias in the cumulative fiscal multiplier often reported as a summary statistic. Section 6 outlines a new nonparametric estimator that remains valid in applications when δ is not close to zero and could replace state-dependent LP estimators in applied work. This estimator yields substantially different government spending multipliers when applied to the baseline model of Ramey and Zubairy (2018), consistent with the simulation evidence of large bias in the LP estimator in Sections 4 and 5. The concluding remarks are in Section 7. The online appendix contains the proofs of the main propositions (Appendix A) and additional theoretical results for a multivariate state-dependent structural VAR model when S_t is exogenous (Appendix B). Appendix c discusses the challenges in generalizing our formal results to models with richer state dependence. Details of the simulation design and further simulation results are contained in online Appendix D and E, respectively.

2 Framework

A useful benchmark for studying the properties of state-dependent local projections is a state-dependent stationary structural VAR data generating process for $z_t \equiv (x_t, y_t)'$ that has been discussed frequently in the literature. It takes the form

$$\begin{cases} x_t = \sum_{j=1}^{p} \alpha_{j,t-1} x_{t-j} + \sum_{j=1}^{p} \delta_{j,t-1} y_{t-j} + \varepsilon_{1t}, \\ \\ y_t = \sum_{j=0}^{p} \beta_{j,t-1} x_{t-j} + \sum_{j=1}^{p} \gamma_{j,t-1} y_{t-j} + \varepsilon_{2t}, \end{cases} \tag{2}$$

where the scalar x_t is assumed to be predetermined with respect to y_t and p denotes the lag order.[3] This process includes several empirically relevant special cases. For example, often x_t is simply a directly observed exogenous shock such as a monetary policy shock or a fiscal policy shock ($\alpha_{j,t-1} = 0$, $\delta_{j,t-1} = 0$, $\forall j$). Alternatively, x_t may be an exogenous serially correlated process ($\delta_{j,t-1} = 0$, $\forall j$). The i.i.d. error term $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ defines the vector of mutually independent structural shocks. The variables y_t and ε_{2t} may be higher dimensional. We abstract from this possibility for now, since allowing for higher dimensions would necessitate the use of matrix notation, as in online Appendix B.

[3]In practice, the order of the lag polynomials may differ, in which case p without loss of generality may be interpreted as the highest lag order.

We are interested in the response over time of y_t to a one-time shock in ε_{1t} in this state-dependent structural VAR model. The variable y_t may be detrended or may be expressed in growth rates, in which case the growth rate response is cumulated. These response functions may also be used to derive multipliers, in which case y_t includes all variables needed to define the multiplier. Identification requires the structural model to be block recursive with respect to y_t.

The model coefficients evolve over time, depending on the state of the economy. Unlike in Markov switching models, the state of the economy is observed. In the simplest and most common case, there are only two states (such as a recession or an expansion). An important feature of the data generating process (1) is how this binary indicator is determined. For example, a recession is often defined as the unemployment rate exceeding some threshold and an expansion as the unemployment rate falling below that threshold. The unemployment rate in this example, in turn, may be included in y_t or not. More generally, the variable defining the state may be exogenous with respect to z_t, although that situation rarely arises in practice.

Thresholds in turn may be exogenously given or may refer to multiples of standard deviations of the variable in question from its mean over the estimation period (or standard deviations from zero, if that variable is always positive). Alternatively, the economy may be in recession if real output is below some trend line and in expansion if it is above this trend line, where the trend line may refer to a two-sided or a one-sided moving average filter or possibly some higher-order deterministic trend. More generally, the states could depend on multiple binary indicators.

2.1 A simple structural model

To illustrate our main results, we focus on a structural data generating process for $z_t = (x_t, y_t)'$ of the form

$$\begin{cases} x_t = \varepsilon_{1t} \\ y_t = \beta_{t-1} x_t + \gamma_{t-1} y_{t-1} + \varepsilon_{2t}, \end{cases} \tag{3}$$

that closely mimics key features assumed in many empirical applications. This process is a special case of data generating process (1) that facilitates the analytical derivation of the limit of the state-dependent LP estimator, allowing us to gain intuition for when the state-dependent LP estimator is expected to be valid and when it is not.

Setting $x_t = \varepsilon_{1t}$ corresponds to the empirically relevant situation of ε_{1t} being identified based on information extraneous to the model. A popular example is the narrative approach to identifying monetary policy shocks (e.g., Romer and Romer (1989), Tenreyro and Thwaites (2016)) and fiscal policy shocks (e.g., Ramey and Shapiro (1998), Ramey (2011), Ramey (2016)).

All model coefficients evolve over time depending on the state of the economy. In the simplest case, there are only two states (such as a recession and an expansion). Accordingly, let $\beta_{t-1} = \beta_E S_{t-1} + \beta_R (1 - S_{t-1})$ and define γ_{t-1} similarly, where S_{t-1} is a binary stationary time series that takes the value $s = 1$ if the economy is in expansion and $s = 0$ otherwise. In particular, we assume that S_t is an observed (binary) deterministic function of elements of $\{w_r = (x_r, y_r, q_r)' : r \leq t\}$, a set which contains the random variables used to construct S_t. These include potentially the endogenous variables in the system $z_t = (x_t, y_t)'$ and their lags, as well as a third variable q_t (and its lags). We assume that q_t is determined outside the model and is strictly exogenous with respect to ε_t. An example of such a variable would be a measure of "animal spirits" or "sentiment" that acts like a sunspot driving the business cycle (e.g., Blanchard (1993), Hall (1993), Barsky and Sims (2012)). For example, Blanchard (1993) suggests that the 1990-91 recession was caused by an exogenous increase in pessimism that caused a sharp reduction in aggregate demand. More specifically, let

$$S_t = \eta (w_r : r \leq t), \tag{4}$$

where $\eta(\cdot)$ is the composition of the indicator function and the function of $\{w_r : r \leq t\}$ used to indicate that S_t equals 1 or 0. For instance, if S_t is 1 whenever $y_t > 0$ and is 0 otherwise, then $S_t = \eta(y_t) \equiv 1(y_t > 0)$, in which case $w_t = y_t$.

We make the following additional assumptions.

Assumption 1 *$\{\varepsilon_{1t}\}$ and $\{\varepsilon_{2t}\}$ are mutually independent structural shocks such that $\varepsilon_t \equiv (\varepsilon_{1t}, \varepsilon_{2t})' \sim i.i.d.(0, \Sigma)$, where Σ is a diagonal matrix with diagonal elements given by σ_i^2 for $i = 1, 2$. In addition, y_t is strictly stationary and ergodic.*

Assumption 2 *$\{q_t\}$ is independent of $\{\varepsilon_{1t}\}$ and $\{\varepsilon_{2t}\}$.*

Assumption 1 is stricter than a martingale difference sequence assumption on ε_t and rules out conditional heteroskedasticity, but is standard in the nonlinear structural VAR literature. In addition, we assume that the process for y_t is strictly stationary and ergodic. Assumption 2 formalizes the idea that q_t is a strictly exogenous process that is determined outside the economic system. For instance, q_t may capture exogenous consumer or investor sentiment. Note that this assumption does not rule out temporal dependence in q_t. We are not assuming that q_t is i.i.d., but only that it is strictly exogenous with respect to model variables such as inflation or output. The main reason we introduce this exogenous random variable q_t is because the limit of the state-dependent LP estimator depends on whether S_t is endogenous or exogenous. The latter case corresponds to setting η as a function of q_t (and its lagged values) only.

Next, we introduce two possible definitions of the impulse response function conditional on the state of the economy.

2.2 Conditional impulse response functions

Our goal is to define the causal effect on y_{t+h} of a one-time shock in ε_{1t}, conditionally on S_{t-1}, the state of the economy at time $t-1$. The latter conditioning set has been standard in the literature on state-dependent LP regressions since Auerbach and Gorodnichenko (2013a,b), who in turn build on the assumptions in the state-dependent structural VAR model in Auerbach and Gorodnichenko (2012). The state dependence of the population process has implications for the definition of the conditional impulse response function. A common approach inspired by the literature on nonlinear impulse response functions (e.g., Gallant, Rossi and Tauchen (1993), Koop, Pesaran and Potter (1996), Potter (2000), Gourieroux and Jasiak (2005), Kilian and Vigfusson (2011), Gonçalves, Herrera, Kilian and Pesavento (2021, 2022)) is to compare, all else equal, two sample paths for the outcome variable of interest, one where ε_{1t} is subject to a one-time shock at time t and another one where no such shock is present. We follow this approach here, but formalize it using a potential outcomes framework. Although the latter approach is common in the microeconometrics literature on treatment effects, it only gradually has gained traction in macroeconometrics (e.g., White (2006), White and Kennedy (2009), Angrist and Kuersteiner (2011), Angrist, Jordà, and Kuersteiner (2018), and most recently Rambachan and Shephard (2021) and Cloyne, Jordà and Taylor (2023)).

A potential outcome model is a model that tells us the observed value of y_{t+h} for any fixed value of ε_{1t}. To distinguish between the random variable ε_{1t} and any fixed value it might take, we denote the latter by e. Thus, if ε_{1t} takes on values in a set A, then $e \in A$. For instance, if ε_{1t} is a binary treatment, we have only two possible values for e, 0 and 1, in which case $A = \{0, 1\}$. In the macroeconomic setting considered here, ε_{1t} is a continuous random variable.

Let $y_{t+h}(e)$ define the potential outcome associated with fixing ε_{1t} at any possible value e in the support of ε_{1t}. When $e = \varepsilon_{1t}$, we obtain the observed value of y_{t+h}, i.e., $y_{t+h}(\varepsilon_{1t}) = y_{t+h}$. Our definition of the conditional impulse response function is based on comparing this baseline value with $y_{t+h}(\varepsilon_{1t} + \delta)$, the counterfactual value of y at $t+h$ that would have been observed if ε_{1t} had been subject to a shock of discrete size δ (see, e.g., Potter (2000)).

Definition 1 (Conditional average response) *The conditional average response function of y_{t+h} to a one-time shock of fixed size δ in ε_{1t} is defined as*

$$CAR_h(\delta, s) = E\left(y_{t+h}(\varepsilon_{1t} + \delta) - y_{t+h}(\varepsilon_{1t}) \,|S_{t-1} = s\right)$$

where $s \in \{0, 1\}$.

In our setup, the potential outcome $y_{t+h}(e)$ is a random variable obtained by solving the structural model (3) and (4) forward, letting $\varepsilon_{1t} = e$. This defines $y_{t+h}(e) = m_h(e, U_{t+h})$, where m_h is a potentially complicated function of e and U_{t+h}, and U_{t+h} contains the structural shocks on the two variables up to time $t + h$, except for ε_{1t} which is set at e, as well as the values of q between $t - 1$ and $t + h - 1$, and the initial condition z_{t-1}. The conditional average response function (CAR) is the expectation of $y_{t+h}(\varepsilon_{1t} + \delta) - y_{t+h}(\varepsilon_{1t})$ with respect to $(\varepsilon_{1t}, U_{t+h})$, conditionally on S_{t-1}. We call this object the conditional average response function by analogy to the notion of a (conditional) average treatment effect.

Definition 1 is similar but not identical to the definition of a conditional nonlinear response function in Gallant, Rossi and Tauchen (1993), Koop, Pesaran and Potter (1996), Potter (2000), and Gonçalves, Herrera, Kilian and Pesavento (2021), among others. One difference is that these studies conditioned on the entire information set known at time $t - 1$ (i.e. the filtration $\mathcal{F}^{t-1}$ generated by the past of (y_t, x_t, q_t) in our framework). Instead, we condition on a much smaller information set, consisting of S_{t-1} only. The reason for conditioning on S_{t-1} only and not on $\mathcal{F}^{t-1}$ is that this corresponds to the empirical practice in the LP literature on estimating state-dependent responses. Notably, applied researchers are interested in the impulse response function of an outcome variable conditional on being in an expansion or in a recession[4]. The second difference is that some studies compare two potential outcomes: $y_{t+h}(e)$ and $y_{t+h}(e')$, where e and e' are fixed. We instead are comparing $y_{t+h}(\varepsilon_{1t} + \delta)$ against $y_{t+h}(\varepsilon_{1t})$, where the latter corresponds to the observed value y_{t+h}, whereas the former denotes the counterfactual value $y_{t+h}(\varepsilon_{1t} + \delta)$ that is not observed. Since ε_{1t} is random, the conditional expectation in Definition 1 averages over all possible realizations of ε_{1t} (in addition to the other sources of randomness that enter into the potential outcomes), conditionally on S_{t-1}.

[4]The reason presumably is that allowing for lagged variables in the conditioning set in general implies that the value of the impulse response function may differ for each trajectory of the past data. Clearly, a definition of the impulse response function that conditions on the entire history would not be something that could ever be consistently estimated with a state-dependent local projection that only estimates two impulse response functions, one for each state at time $t - 1$.

Another possible definition of a conditional impulse response function treats the size of the shock as infinitesimal. This corresponds to a conditional marginal treatment effect in microeconometrics and for this reason will be referred to as a conditional marginal response (CMR).

Definition 2 (Conditional marginal response) *The conditional marginal response function of y_{t+h} to an infinitesimal shock in ε_{1t} is defined as*

$$CMR_h(s) = \lim_{\delta \to 0} \frac{CAR_h(\delta, s)}{\delta}$$

for any value of $s \in \{0, 1\}$.

Definition 2 mirrors the definition proposed in Potter (2000), except for the fact that we condition on S_{t-1}. Defining the conditional marginal response as the limit of CAR_h, scaled by δ, accommodates models with potential outcome functions that are not differentiable, such as the models considered in this paper, which involve states defined by an indicator function.

Although considering the effect of an infinitesimal shock on the outcome variable of interest is not as popular in state-dependent models as considering the effect of a shock of fixed magnitude δ, we consider both the conditional marginal and conditional average response functions because local projection estimands may relate to either of these two definitions.[5] In particular, as we will show next, state-dependent local projections identify the conditional marginal response function when S_t is exogenous under Assumptions 1 and 2. Under these assumptions, the LP estimand is also equal to the conditional average response function for $\delta = 1$. In contrast, when S_t is endogenous, the state-dependent LP estimator is inconsistent for the conditional average response function, although it may still recover the conditional marginal response function under additional assumptions.

3 State-dependent local projections

Recall that a state-dependent local projection estimates the dynamic effect of ε_{1t} on y_{t+h} conditionally on the state of the economy by a set of regressions for each horizon h:

$$y_{t+h} = S_{t-1}\left[b_h(1)\,\varepsilon_{1t} + \pi'_{E,h} z_{t-1}\right] + (1 - S_{t-1})\left[b_h(0)\,\varepsilon_{1t} + \pi'_{R,h} z_{t-1}\right] + v_{t+h}, \quad (5)$$

[5]This distinction is not needed when the model is linear in the potential outcome as both definitions coincide.

where S_{t-1} takes the value 1 if the economy is in expansion and 0 if it is in recession. The least squares estimate $\hat{b}_h(1)$ of the slope coefficient associated with $\varepsilon_{1t} S_{t-1}$ is usually interpreted as the impulse response of y_{t+h}, conditionally on $S_{t-1} = 1$, whereas $\hat{b}_h(0)$ is interpreted as the response of y_{t+h} when conditioning on $S_{t-1} = 0$. The main goal of this section is to clarify the interpretation of this LP estimator and the conditions required for a causal interpretation of the estimates. Specifically, we derive the probability limits of $\hat{b}_h(1)$ and $\hat{b}_h(0)$ and relate these estimands to the two definitions of the conditional impulse response function given above. We derive results for two scenarios: one in which S_t is exogenous and another one in which S_t is endogenous with respect to ε_t.

3.1 Exogenous S_t

We assume that (4) holds with $w_r = q_r$ such that $S_t = \eta(q_r : r \leq t)$. This corresponds to the case of S_t being constructed using only information on the variable q_t. Given Assumption 2, this implies that S_t is exogenous with respect to ε_{1t} and ε_{2t}.

We first derive the conditional impulse response functions $CAR_h(\delta, s)$ and $CMR_h(s)$ for this model. Under our assumptions, the potential outcome model $y_{t+h}(e)$ is

$$y_{t+h}(e) = (\gamma_{t+h-1} \cdots \gamma_t)\, \beta_{t-1} e + V_{t+h} \equiv m_h(e, U_{t+h}), \tag{6}$$

where $(\gamma_{t+h-1} \cdots \gamma_t)$ and V_{t+h} are both functions of $U_{t+h} \equiv (\varepsilon_{t+h}, \ldots, \varepsilon_{t+1}, \varepsilon_{2t}, q_{t+h-1}, \ldots, q_{t-1}, z_{t-1})'$. This follows easily by iterating on the model equation for y_t in (3) fixing $\varepsilon_{1t} = e$ and using the fact that S_t is exogenous (and, hence, not affected by e). Thus, for any e, m_h is a linear function of e and

$$y_{t+h}(e + \delta) - y_{t+h}(e) = [(\gamma_{t+h-1} \cdots \gamma_t)\, \beta_{t-1}]\, \delta.$$

This implies that

$$CAR_h(\delta, s) = E\left[(\gamma_{t+h-1} \cdots \gamma_t)\, \beta_{t-1} | S_{t-1} = s\right] \delta,$$

and

$$\frac{CAR_h(\delta, s)}{\delta} = CMR_h(s) = E\left[(\gamma_{t+h-1} \cdots \gamma_t)\, \beta_{t-1} | S_{t-1} = s\right].$$

This is true for any fixed δ and also for $\delta \to 0$. This shows that, when the state of the economy is exogenous, the conditional marginal response function coincides with the conditional average response function $CAR_h(\delta, s)$ for a shock of size $\delta = 1$.

We summarize these results in the next proposition. Let $\beta_s = \beta_E$ if $s = 1$ and $\beta_s = \beta_R$ if

$s = 0$.

Proposition 3.1 *Assume the structural process is (3) and (4) with* $S_t = \eta(q_s : s \leq t)$. *Under Assumptions 1 and 2, the following results hold for* $s \in \{0, 1\}$*:*

(i) *For any fixed* δ, $CAR_0(\delta, s) = \beta_s \delta$, *and for any* $h \geq 1$,

$$CAR_h(\delta, s) = E[\gamma_{t+h-1} \cdots \gamma_t | S_{t-1} = s] \beta_s \delta.$$

(ii) $CMR_0(s) = \beta_s$, *and for any* $h \geq 1$,

$$CMR_h(s) = E[\gamma_{t+h-1} \cdots \gamma_t | S_{t-1} = s] \beta_s.$$

Part (i) of Proposition 3.1 gives the conditional average response function for any shock of size δ, where δ is fixed. For $h = 0$, the impact response function of y_t to a shock of size $\delta = 1$ in ε_{1t} is β_s, which is either β_E or β_R depending on whether we were in an expansion ($s = 1$) or in a recession ($s = 0$) prior to the shock. For longer horizons, the conditional average response function depends on the state of the economy at time $t - 1$, but not on the current or future states of the economy. Nor do we condition on the history of states prior to $t - 1$. Rather, we average them out and condition only on the most recent state. This allows us to examine how the impulse response function differs depending on whether the economy was in expansion or recession prior to the shock. Part (ii) of Proposition 3.1 gives the conditional marginal response function in Definition 2. This response function traces the dynamic causal effect of an infinitesimal shock in ε_{1t} on y_{t+h}.

Next, we derive the probability limits of the state-dependent LP estimates $\hat{b}_h(1)$ and $\hat{b}_h(0)$. We can obtain each of these separately, by restricting the sample to $S_{t-1} = 1$ and $S_{t-1} = 0$, respectively. For instance, $\hat{b}_h(1)$ can be obtained by a regression of y_{t+h} on $\varepsilon_{1t} S_{t-1}$ and $z_{t-1} S_{t-1}$ (omitting $\varepsilon_{1t}(1 - S_{t-1})$ and $z_{t-1}(1 - S_{t-1})$ from the regression). This follows because $S_{t-1}(1 - S_{t-1}) = 0$ for all t.

Under the assumed stationarity and ergodicity of ε_{1t} and y_t, it can be shown easily that

$$\hat{b}_h(s) \to_p b_h(s) = \frac{E(y_{t+h} \varepsilon_{1t} | S_{t-1} = s)}{E(\varepsilon_{1t}^2 | S_{t-1} = s)},$$

where the LP estimand $b_h(s)$ can be interpreted as the population OLS coefficient associated with ε_{1t} in a linear regression of y_{t+h} on ε_{1t} which conditions on $S_{t-1} = s$.

Proposition 3.2 *Consider the structural process (3) and (4) with* $S_t = \eta(q_r : r \leq t)$. *If Assumptions 1 and 2 hold, then for* $s \in \{0, 1\}$,

$$b_h(s) = CMR_h(s) = \frac{CAR_h(\delta, s)}{\delta} = CAR_h(1, s).$$

The main implication of Proposition 3.2 is that state-dependent local projections are valid when S_t is strictly exogenous with respect to the structural shocks in the model, as would be the case when q_t represents an exogenous measure of sentiment. Under these conditions, $b_h(s)$ is equal to the conditional marginal response function, which gives the effect of an infinitesimal size shock in ε_{1t} on y_{t+h}. However, Proposition 3.2 also shows that another valid interpretation of $b_h(s)$ is that it gives the conditional average response of y_{t+h} to a shock of fixed size δ, in proportion to δ. When $\delta = 1$, $b_h(s)$ captures the conditional average effect of a shock of size 1 in ε_{1t} on y_{t+h}. Both interpretations are correct and coincide with each other under the assumed exogeneity of S_t.

Model (3) helps illustrate why the exogeneity of S_t is important for deriving this result. To see this, condition on the economy being in expansion such that $s = 1$. At horizon $h = 0$, the model implies that

$$y_t = \beta_E \varepsilon_{1t} + \underbrace{\gamma_E y_{t-1} + \varepsilon_{2t}}_{=v_t},$$

where the underlying error term v_t is independent of ε_{1t} under Assumptions 1 and 2, conditionally on $S_{t-1} = 1$. Thus, the state-dependent LP estimand is $b_0(1) = \beta_E$ and the LP regression recovers both of the conditional average and marginal impulse response functions on impact.

At horizon $h = 1$, conditionally on $S_{t-1} = 1$,

$$y_{t+1} = \beta_t \varepsilon_{1t+1} + \gamma_t (\beta_E \varepsilon_{1t} + \gamma_E y_{t-1} + \varepsilon_{2t}) + \varepsilon_{2t+1} = \gamma_t \beta_E \varepsilon_{1t} + v_{t+1},$$

where $v_{t+1} = \gamma_t \gamma_E y_{t-1} + \gamma_t \varepsilon_{2t} + \beta_t \varepsilon_{1t+1} + \varepsilon_{2t+1}$. This model has a heterogeneous slope coefficient $\gamma_t \beta_E$ because γ_t is a function of the state indicator S_t. The regression of y_{t+1} on ε_{1t} recovers the conditional expectation of $\gamma_t \beta_E$, conditionally on $S_{t-1} = 1$, provided v_{t+1} is conditionally independent of ε_{1t}. Setting $S_t = \eta(q_r : r \leq t)$ with q_r satisfying Assumption 2, the LP estimand for $h = 1$ reduces to

$$b_1(1) = E(\gamma_t | S_{t-1} = 1) \beta_E.$$

For general values of h, we can write y_{t+h} as a function of ε_{1t} and an error term that depends on $S_{t+h-1}, \ldots, S_{t-1}$. Conditionally on S_{t-1}, this equation is state-dependent, as it depends on

$S_{t+h-1}, \ldots, S_t$. A linear local projection of y_{t+h} on ε_{1t} which conditions only on $S_{t-1} = 1$ recovers the conditional average and marginal response functions provided the error term is orthogonal to ε_{1t}, conditionally on S_{t-1}. Since this error depends on $S_{t+h-1}, \ldots, S_t$, we require that ε_{1t} be independent of $S_{t+h-1}, \ldots, S_t$, conditionally on S_{t-1}. This independence condition holds under the assumption that $S_t = \eta(q_r : r \leq t)$ and the process q_t is independent of ε_t, as assumed in Assumption 2.[6]

While this result was derived in the simplest possible setting, in online Appendix B we show that, when S_t is exogenous, the validity of the state-dependent LP estimator generalizes to a multivariate state-dependent structural VAR process $z_t = (x_t, y_t')'$, where y_t is an $n \times 1$ vector of endogenous variables and x_t is predetermined with respect to y_t.

3.2 Endogenous S_t

We now characterize the state-dependent LP estimand when S_t is endogenous with respect to the structural shocks of the system ε_t. To facilitate the derivation of analytical results, we add the following assumption.

Assumption 3 *(a)* $S_t = \eta(\varepsilon_{1t}) = 1(\varepsilon_{1t} > c)$, *where* c *is any constant.* *(b)* $\varepsilon_{1t} \sim$ *i.i.d.* $N(0, \sigma_1^2)$.

Although assuming that S_t is a function of the outcome variable y_t would be more common, we focus on a simpler process in which S_t depends on ε_{1t} because this greatly simplifies the mathematical derivations.[7] This assumption is not without precedent in empirical work. For example, Ben Zeev, Ramey and Zubairy (2023) and Tenreyro and Twhaites (2016) allow the state to depend on the fiscal policy shock or monetary policy shock, respectively. We provide a discussion of the challenges of generalizing this assumption in Section 3.3.

In order to derive the conditional (average and marginal) impulse response functions in this model, we first derive the potential outcomes $y_{t+h}(e)$. As previously, they are obtained by iterating forward the equation for y_t given in (3), fixing $\varepsilon_{1t} = e$. In contrast to the earlier case in which S_t was exogenous, the endogeneity of S_t creates a nonlinearity in the function m_h that defines $y_{t+h}(e) = m_h(e, U_{t+h})$. In particular, for $h = 0$, $y_t(e) = \beta_{t-1} e + V_{0t}$, where

[6] A milder sufficient assumption would have been that the conditional first two moments of ε_t are independent of $S_{t+h-1}, \ldots, S_t$, conditionally on the available information at time $t-1$.

[7] In particular, under Assumption 3(a) S_t is i.i.d. since ε_{1t} is i.i.d. This implies that a shock to ε_{1t} only impacts the date t coefficients in the model for y_t. Nevertheless, all the conditional impulse response functions are affected by this shock. The Gaussianity assumption on ε_{1t} is also used to obtain the closed form expression for the local projection estimands in this setting.

$V_{0t} = \gamma_{t-1} y_{t-1} + \varepsilon_{2t}$ is a function of $U_t = (\varepsilon_{2t}, z_{t-1})$. This is still a linear function of e, as was the case for S_t exogenous. However, for horizon $h = 1$, we now obtain that

$$y_{t+1}(e) = \gamma(e)\beta_{t-1}e + V_{t+1}(e) \equiv m_1(e, U_{t+1}),$$

where $\gamma(e) \equiv \gamma_R + (\gamma_E - \gamma_R)\eta(e)$, with $\eta(e) = 1(e > c)$ a nonlinear function of e, and $V_{t+1}(e) = \gamma(e)V_{0t} + \beta(e)\varepsilon_{1t+1} + \varepsilon_{2t+1} \equiv V_1(e, U_{t+1})$ is a nonlinear function of e and $U_{t+1} = (\varepsilon_{t+1}, \varepsilon_{2t}, z_{t-1})'$. More generally, for any $h > 1$, it can be shown that

$$y_{t+h}(e) = [\gamma_{t+h-1} \cdots \gamma_{t+1}\gamma(e)]\beta_{t-1}e + V_{t+h}(e) \equiv m_h(e, U_{t+h}), \tag{7}$$

where m_h is a nonlinear function of e and $U_{t+h} \equiv (\varepsilon_{t+h}, \ldots, \varepsilon_{t+1}, \varepsilon_{2t}, z_{t-1})'$.

There are three important differences from the potential outcome model (6) derived under the assumption of exogenous S_t. First, given that $\gamma(e)$ depends on $\eta(e) = 1(e > c)$ which is a nonlinear function of e, the first term of (7) is nonlinear in e. Second, as shown in online Appendix A, the term $V_{t+h}(e)$ is also a nonlinear function of e (whereas V_{t+h} did not depend on e in (6)). This makes $y_{t+h}(e)$ a nonlinear function of e. Third, $y_{t+h}(e)$ is non-differentiable since the state is determined by an indicator function. This prevents us from defining the conditional marginal response in terms of the first derivative of the potential outcome, as in Rambachan and Shephard (2021).

Using the potential outcomes (7), we obtain the following result. As before, let $\beta_s = \beta_E$ if $s = 1$ and $\beta_s = \beta_R$ if $s = 0$, and define γ_s similarly. Also let $\bar{\gamma} \equiv E(\gamma_t) = \gamma_R + (\gamma_E - \gamma_R)\Phi(-c/\sigma_1)$ and $\upsilon_s \equiv \gamma_s E(y_{t-1}|S_{t-1} = s)$, where we let $\Phi(\cdot)$ and $\phi(\cdot)$ denote the cumulative density function (cdf) and probability density function (pdf) of a standard normal distribution.

Proposition 3.3 *Assume the structural process is (3) and (4) with $S_t = \eta(\varepsilon_{1t})$. Under Assumptions 1 and 3, we have that for $s \in \{0, 1\}$:*

(i) *$CAR_0(\delta, s) = \beta_s\delta$, and for $h \geq 1$, $CAR_h(\delta, s) = (\bar{\gamma})^{h-1} CAR_1(\delta, s)$, where*

$$\begin{aligned} CAR_1(\delta, s) \quad = \quad & \{\gamma_R + (\gamma_E - \gamma_R)\Phi(-c/\sigma_1)\}\beta_s\delta \\ & + \{\gamma_R + (\gamma_E - \gamma_R)[\Phi(-c/\sigma_1 + \delta/\sigma_1) - \Phi(-c/\sigma_1)]\}\beta_s\delta \\ & + \{(\gamma_E - \gamma_R)\sigma_1[\phi(-c/\sigma_1 + \delta/\sigma_1) - \phi(-c/\sigma_1)]\}\beta_s \\ & + \{(\gamma_E - \gamma_R)[\Phi(-c/\sigma_1 + \delta/\sigma_1) - \Phi(-c/\sigma_1)]\}\upsilon_s. \end{aligned}$$

(ii) *$CMR_0(s) = \beta_s$, and for any $h \geq 1$, $CMR_h(s) = (\bar{\gamma})^{h-1} CMR_1(s)$, where*

$$CMR_1(s) = \{\gamma_R + (\gamma_E - \gamma_R)\,\Phi(-c/\sigma_1)\}\beta_s + \{(\gamma_E - \gamma_R)\,\sigma_1^{-1}\phi(c/\sigma_1)\}\,(c\beta_s + v_s).$$

As the proof of Proposition 3.3 in the online Appendix A reveals, we can decompose $CAR_1(\delta, s)$ into the sum of a direct effect and an indirect effect given by

$$\begin{aligned} \text{Direct effect} &= E(\gamma(\varepsilon_{1t}))\beta_s\delta \\ \text{Indirect effect} &= E\left[(\gamma(\varepsilon_{1t}+\delta) - \gamma(\varepsilon_{1t}))\right](\beta_s\delta + v_s) + E\left[(\gamma(\varepsilon_{1t}+\delta) - \gamma(\varepsilon_{1t}))\,\varepsilon_{1t}\right]\beta_s. \end{aligned}$$

The direct effect can also be written as $E(\gamma_t)\,\beta_s\delta$ (since $\gamma(\varepsilon_{1t}) = \gamma_t$), which coincides with what was derived in Proposition 3.1 for exogenous S_t. This direct effect captures the effect of a change in ε_{1t} on y_{t+h} that keeps γ_t constant, as would have been the case if S_t had been exogenous. However, in the current model, $S_t = \eta(\varepsilon_{1t})$. Thus, perturbing ε_{1t} by δ also has an impact on the model parameters at time t. This indirect effect depends on the difference between $\gamma(\varepsilon_{1t}+\delta)$ and $\gamma(\varepsilon_{1t})$.

Being able to derive an analytical expression for the indirect effect in this setting helps illustrate three important points. First, the larger the difference between the parameters in the two states, the larger the indirect effect will be. Second, the indirect effect will be larger, the higher the probability of entering a different state, given δ. Third, the smaller the ratio of δ over σ_1, the smaller the indirect effect. The simulations in the next section quantify how important it is for practitioners to think about the size of the shock relative to the standard deviation of ε_{1t}.

The expressions provided in Proposition 3.3 are the result of evaluating these expectations under the Gaussianity assumption on ε_{1t}. Note that part (ii) of Proposition 3.3 follows from part (i) by considering the limit of $\delta^{-1}CAR_h(\delta, s)$ as $\delta \to 0$. More generally, Proposition 3.3 shows that the conditional average response function no longer coincides with the conditional marginal response function when S_t is endogenous.

Next, we derive the state-dependent local projection estimands $b_h(s)$, as implied by equation (5), and show that they coincide with $CMR_h(s)$.

Proposition 3.4 *Assume the structural model is (3) and (4) where $S_t = \eta(\varepsilon_{1t})$. Under Assumptions 1 and 3, we have that for $s \in \{0, 1\}$, for any $h \geq 0$, $b_h(s) = CMR_h(s)$.*

Given Proposition 3.3, Proposition 3.4 has two main implications. First, the state-dependent

LP estimator is consistent for the conditional marginal response function at all horizons $h \geq 0$. This holds under Assumptions 1 and 3, which allows S_t to be endogenous of the form $S_t = 1(\varepsilon_{1t} > c)$, where $\varepsilon_{1t} \sim N(0, \sigma_1^2)$. Thus, the state-dependent LP estimand can be interpreted as giving the effect of an infinitesimally sized shock ε_{1t} on y_{t+h} under these simplifying assumptions. The second implication is that, in general, the state-dependent LP does not recover the conditional average response function. In particular, the asymptotic bias of LP, when the target impulse response function is given by the conditional average response for $\delta = 1$, is the difference between $CMR_h(s)$ in part (ii) of Proposition 3.3 and $CAR_h(1, s)$. More generally, LP does not consistently estimate the dynamic causal effect of a perturbation by a fixed δ in ε_{1t} on y_{t+h}.[8]

4 Which conditional impulse response function is more relevant for applied work?

When S_t is endogenous, as is typically the case in practice, the researcher needs to decide whether the object of interest is the conditional average response or the conditional marginal response. If the latter is the object of interest, state-dependent LP estimators provide an easy and consistent way of estimating this impulse response. In contrast, if the researcher is interested in estimating the effect of a shock of fixed size δ, consistent estimation of the conditional average response is required. Ultimately, this choice should be guided by the research question the practitioner is interested in.

4.1 Implications for the state-dependent LP estimator

Despite the importance of the value of δ for the accuracy of impulse response analysis, this parameter is rarely, if ever, discussed in applied work. Often users interpret responses estimated by state-dependent local projections as the effects of a $\delta = 1$ shock. Sometimes users rescale these response estimates to represent responses to smaller or larger shocks. How biased these response estimators are in our setting, depends on the variance of ε_{1t}, denoted σ_1^2, which can

[8] It should also be noted that the LP estimator cannot be reinterpreted as an estimator of the direct effect, unless S_t is exogenous. When S_t is endogenous, as shown in Proposition 3.4, the LP estimator captures the conditional marginal response, but not the conditional average response derived in part (i) of Proposition 3.3. As shown in part (ii) of Proposition 3.3, the conditional marginal response consists of the direct effect, corresponding to the first term in $CMR_1(s)$ plus an additional component captured by the second term of the $CMR_1(s)$. This component corresponds to the limit as $\delta \to 0$ of the three last terms in the $CAR_1(\delta, s)$, which define the indirect effect. Thus, the LP estimator does not fully capture the indirect effect unless $\delta \to 0$.

be estimated from the data. It can be shown that, in empirical applications involving fiscal and monetary policy shocks, σ_1 may range from about 0.6 (e.g., Tenreyro and Thwaites (2016)) to as low as 0.05 (e.g., Miranda-Agrippino and Ricco (2021), Gertler and Karadi (2015), Ramey and Zubairy (2018)). Depending on the values of δ and σ_1, the asymptotic bias of the LP estimator can be substantial.

We illustrate this point based on the following data generating process (DGP)

$$\begin{aligned} x_t &= \rho x_{t-1} + \varepsilon_{1t} \\ y_t &= \beta_{t-1} x_t + \alpha_{t-1} x_{t-1} + \gamma_{t-1} y_{t-1} + \varepsilon_{2t}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} \alpha_{t-1} &= \alpha_E S_{t-1} + \alpha_R (1 - S_{t-1}) \\ \beta_{t-1} &= \beta_E S_{t-1} + \beta_R (1 - S_{t-1}), \\ \gamma_{t-1} &= \gamma_E S_{t-1} + \gamma_R (1 - S_{t-1}), \end{aligned} \tag{9}$$

$\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})' \sim N(0, \Sigma)$, Σ is a diagonal matrix with σ_1^2 and σ_2^2 on the main diagonal, and S_t is an indicator for the state of the economy. For expository purposes, we set $\delta = 1$, as is common in empirical applications. Furthermore, we set $\sigma_2 = 1$ and $\sigma_1 = 0.1$, which implies a ratio δ/σ_1 within the range found in applied work. When $S_t = 1$, the economy is in expansion, and, when $S_t = 0$, the economy is in recession. The state of the economy is endogenously determined as $S_t = 1(y_t > 0)$. We consider two special cases of this process. DGP 1 sets $\rho = 0$ such that x_t is a directly observed i.i.d. shock, as is often the case in applied work. Furthermore, we set $\beta_E = 2.5$, $\beta_R = 3.5$, $\gamma_E = 0.7$, $\gamma_R = 0.1$ and $\alpha_{t-1} = 0$. In DGP 2, x_t follows an AR(1) process with $\rho = 0.8$, motivated by the analysis in Alloza, Gonzalo and Sanz (2021), and $\alpha_{t-1} \neq 0$, given $\alpha_E = 1.2$ and $\alpha_R = 0.9$. The population response is evaluated as $CAR_h(\delta, s) = E\left(y_{t+h}(\varepsilon_{1t} + \delta) - y_{t+h}(\varepsilon_{1t})\,|S_{t-1} = s\right)$, whereas the LP estimands are evaluated as $b_h(1) = \frac{E(x_t S_{t-1} y_{t+h})}{E\left(x_t^2 S_{t-1}\right)}$ and $b_h(0) = \frac{E(x_t(1-S_{t-1}) y_{t+h})}{E\left(x_t^2(1-S_{t-1})\right)}$. The number of draws used to compute these conditional expectations is 50 million, which ensures that the LP impact response matches the population response.

Figure 1 illustrates that in this setting the accuracy of the state-dependent LP estimator can be very low. The asymptotic bias of the LP estimator may be as large as 65 percent (expressed in percent deviations from the population response) conditional on a recession and as large as 40 percent conditional on an expansion. Thus, applied users need to consider the implications of the sample standard deviation of the shocks for the accuracy of the LP estimator.

Rather than exploring the role of alternative σ_1 values, given δ, in the remainder of Section 4 we treat σ_1 as a fixed population parameter and allow δ to vary. The advantage of varying δ for fixed σ_1 is that the DGP remains unchanged, allowing a clean assessment of the role of δ. This approach also matches the thought experiment underlying the asymptotic analysis in Section 3.

4.2 Why large δ are relevant for applied work

The central question in assessing the magnitude of the asymptotic bias documented in Section 3 is which values of δ are empirically relevant for a given σ_1. On the one hand, there may be applications in which a small δ relative to σ_1 is of interest. For example, even granting that monetary policy shocks that exceed one sample standard deviation are not uncommon, there may be interest in the effects of a more modest 25 basis points monetary policy shock. Even such a small shock, however, would translate to a δ that is 84 percent of one standard deviation of commonly used measures of exogenous monetary policy shocks.[9]

On the other hand, there are applications that call for much larger δ relative to σ_1. For example, as stressed by Ramey and Zubairy (2018, p. 853-854), estimating the effects of government spending purchases on the economy requires "the identified changes in government spending to be not only exogenous, but big enough for their effects to be extracted from the many other economic shocks hitting the economy". We can infer what they mean by "big" because they single out World War I, World War II and the Korean War as the three most important examples of such big spending shocks in their data from 1890Q1 to 2015Q4. The World War II spending shock, for example, is 12 times the standard deviation of their military spending shock. Similarly, in the post-war quarterly data, the shock associated with the Korean War in 1950Q3 is about thirteen times the sample standard deviation of the military spending shock, giving us an idea of the relevant range of δ/σ_1.

These examples show that the case of large δ relative to σ_1 is empirically relevant, even if it may not be the only case applied users are interested in. Our objective is to help researchers understand the econometric implications of this point. Next, we illustrate the magnitude of the

[9] A case in point is the narrative monetary policy shock series of Romer and Romer (2004), as updated by Wieland and Yang (2020). Given data from January 1969 to December 2007, the monthly series of monetary policy shocks has a standard deviation of 0.296. Policy shocks range from -3.25 in April 1980 to 1.86 in November 1980, which implies magnitudes of δ between -11 and 6 standard deviations. After aggregating these data to quarterly frequency, the magnitude of the policy shocks ranges from -7 to 4 standard deviations.

asymptotic bias of the state-dependent LP estimator of the average treatment effect by varying δ for a given σ_1. Later we also examine how this bias affects estimates of the multiplier. Our aim is not to provide a comprehensive simulation analysis of the accuracy of state-dependent local projections, but to show that the bias in the estimates can be substantial in empirically relevant settings.

4.3 How much does the choice of δ matter?

The empirical relevance of large values of δ relative to σ_1 motivates a closer look at the mechanism that makes state-dependent local projections fail when δ is large and S_t is endogenous. Intuitively, given the analysis in Section 3, one would expect the LP estimator to provide a good approximation as long as the economy remains in the same state following a shock. For given σ_1, this is more likely to be the case when δ is close to zero. Figure 2 illustrates this point for $\delta \in \{0.25, 1, 5, 10\}$ while setting $\sigma_1 = 1$. Otherwise, the DGPs match those described in Section 4.1. Figure 2 shows the asymptotic bias of the impulse responses obtained by state-dependent local projections (expressed in percent deviations from the population response).[10] The values of δ have been chosen to imply ratios δ/σ_1 representative of empirical work. Even larger ratios could be considered. For example, Ramey and Zubairy (2018) estimate the response of real GDP to a military spending shock based on a state-dependent local projection with $\delta/\sigma_1 \approx 17$, given $\delta = 1$.[11]

Figure 2 confirms that the size of the asymptotic bias at longer horizons declines as $\delta \to 0$, but can be substantial for large δ. The bias can be as large as 82 percent for $\delta = 5$ and as large as 23 percent for $\delta = 0.25$. This result is qualitatively robust to the use of alternative DGPs.

Further insight may be gained by decomposing $CAR(\delta, s)$ into the direct effect of the shock and the indirect effect (obtained as the difference between the population response and the direct effect). Figure 3 shows this decomposition for $\delta = 5$ in DGP 1 and DGP 2. It is readily apparent that the LP estimator differs from the direct effect when S_t is endogenous, and that it does not fully capture the indirect effect of perturbing ε_{1t} by δ on the model parameters, as

[10] The state-dependent LP estimator implicitly sets the shock size δ to unity, which in general differs from one standard deviation of the structural error postulated in this exercise. However, given that δ in our DGP is a multiple of the standard deviation of ε_{1t}, LP responses to δ standard deviations are constructed by scaling the estimated LP response function by a factor of δ as is customary in applied work.

[11] The defense spending news series originally constructed by Ramey (2011) and then expanded by Ramey and Zubairy (2018) is defined as the change in the expected present discounted value of government spending for events that were related to political or military events. The nominal value of these changes are then divided by nominal GDP lagged one quarter. Setting $\delta = 1$ thus amounts to postulating a fiscal shock that corresponds to one percent of GDP.

suggested by the analysis in Section 3.2. Since larger positive δ values make it more likely that a policy shock would catapult the economy from recession to expansion, conditional on being in a recession, the LP estimator becomes increasingly inaccurate. Additional simulation results in online Appendix D show that the magnitude of this indirect effect, and hence the asymptotic bias, increases with $\gamma_E - \gamma_R$ and could be even larger than in the examples shown here.

5 Revisiting the government spending multiplier

What matters from an applied point of view is not only that the state-dependent LP estimator is inconsistent when S_t is endogenous and δ is nonnegligible, as we have shown, or how large the asymptotic bias of the LP response estimator is, but also how large the bias of the corresponding cumulative multipliers is. In this section, we quantify this bias for $\delta \in \{1, 5, 10\}$, given a prototypical data generating process motivated by Ramey and Zubairy's (2018) widely cited study of the government spending multiplier in good times and bad times. Given that Ramey and Zubairy employed state-dependent local projections, they did not specify the underlying DGP, but the following process is one empirically plausible representation of such DGP.

Let $z_t = (x_t, g_t, y_t)'$ where x_t denotes Ramey and Zubairy's military spending news measure, g_t is real government spending, and y_t is real GDP in period t, all relative to potential GDP in period $t-1$. The data are quarterly. Consider a trivariate state-dependent data generating process given by

$$C_{t-1} \begin{bmatrix} x_t \\ g_t \\ y_t \end{bmatrix} = k_{t-1} + B_{t-1}(L) \begin{bmatrix} x_{t-1} \\ g_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \tag{10}$$

where

$$\begin{aligned} k_{t-1} &= S_{t-1} k_E + (1 - S_{t-1})\, k_R, \\ C_{t-1} &= S_{t-1} C_E + (1 - S_{t-1})\, C_R, \\ B_{t-1}(L) &= S_{t-1} B_E(L) + (1 - S_{t-1})\, B_R(L), \end{aligned} \tag{11}$$

ε_t is a vector of mutually independent $N(0,1)$ population innovations, $B_E(L)$ and $B_R(L)$ are lagged polynomials, and the coefficient matrices are of suitable dimensions. Thus, $\delta = 1$ corresponds to a one standard-deviation shock. The model allows for four autoregressive lags. We obtain the parameter values by fitting the model to the historical data used by Ramey and Zubairy (2018). To conserve space, the parameter values used in the simulation study are reported in online Appendix C.

Ramey and Zubairy (2018) explored several alternative definitions of economic expansions including periods when the unemployment rate is below the sample mean, positive deviations from the Hodrick-Prescott trend of the unemployment rate, and expansion dates determined by the NBER business cycle committee.[12] Here we consider two expansion indicators for expository purposes. In one DGP, expansions are defined as periods when output exceeds potential output in period $t-1$,

$$S_t = \begin{cases} 1 \text{ if } y_t > 1 \\ 0 \quad \text{otherwise,} \end{cases}$$

whereas the other DGP defines expansions as periods where real GDP in t, relative to potential GDP in period $t-1$, exceeds a twelve-quarter trailing average,

$$S_t = \begin{cases} 1 \text{ if } y_t > \frac{1}{12}\sum_{i=1}^{12} y_{t-i} \\ 0 \quad \text{otherwise.} \end{cases} .$$

These definitions are similar to those used in Alloza (2022), for example. We further assume that x_t follows an $AR(4)$ process to account for serial correlation in Ramey and Zubairy's military spending news measure (see Alloza, Gonzalo and Sanz (2021)).

Consider the dynamic effects of a shock of magnitude δ in ε_{1t} on g_{t+h} and y_{t+h}. Recall that the population impulse response is defined as $CAR_{g,i}(\delta, s) = E\left(g_{t+i}(\varepsilon_{1t}+\delta) - g_{t+i}(\varepsilon_{1t})\,|S_{t-1}=s\right)$ and $CAR_{y,i}(\delta, s) = E\left(y_{t+i}(\varepsilon_{1t}+\delta) - y_{t+i}(\varepsilon_{1t})\,|S_{t-1}=s\right)$, whereas the LP estimands $b_{g,i}(s)$ and $b_{y,i}(s)$ are defined as previously. In this section, we follow Ramey and Zubairy (2018) in focusing on the implied fiscal multipliers, defined as the relative response of output and government spending to a military spending news shock. The cumulative fiscal multiplier over the h-quarter horizon is defined as

$$\mathcal{M}_h(\delta, s) = \frac{\sum_{i=0}^{h} CAR_{y,i}(\delta, s)}{\sum_{i=0}^{h} CAR_{g,i}(\delta, s)}, \tag{12}$$

in population, whereas the corresponding fiscal multiplier based on the LP estimates is

$$M_h(s) = \frac{\sum_{i=0}^{h} b_{y,i}(s)}{\sum_{i=0}^{h} b_{g,i}(s)}. \tag{13}$$

The number of draws used to compute these conditional expectations is 120 million to ensure

[12]Since the NBER business cycle dates are based on data that are correlated with the endogenous model variables, this approach does not avoid the concerns discussed in Section 3.

that the multiplier on impact matches the population multiplier.

Figure 4 shows results for S_{t-1} depending on the deviation from potential output in the previous period, while Figure 5 shows results for S_{t-1} defined as a function of the one-sided $MA(12)$ filter. The plots illustrate that, at horizon $h = 0$, the LP estimator of the fiscal multiplier recovers the population response for all values of δ, even when S_t is endogenous, consistent with the results in Section 3. However, for larger h, the LP estimator diverges from the population multiplier, especially when the economy is in recession. There is clear evidence of large asymptotic bias (expressed in percentage deviations from the population cumulative multiplier) that is increasing in δ, reaching 45 percent in some cases. In general, the extent of the asymptotic bias depends on the parameter values, the functional form of S_t, the magnitude and sign of the shock δ, and the state of the economy at the time of the policy shock.

Applied researchers most commonly report the two-year and four-year cumulative fiscal multiplier. Table 2 summarizes, for each DGP, the asymptotic bias of these cumulative multipliers for various δ. Whereas for $\delta = 1$ the asymptotic bias of the LP estimator is at most 4% in absolute terms, the bias tends to increase with the magnitude of δ, especially when the economy is in a recession at the time of the government spending shock. When $S_t = 1(y_t > 1)$, given a 5 standard deviation shock, the asymptotic bias of the LP estimator is 18% for the four-year cumulative multiplier, conditional on being in a recession, and 7% conditional on the economy being in expansion. Broadly similar results are obtained for $S_t = 1(y_t > MA(12))$. For example, the bias of the LP estimator of the four year integral is as large as 11% conditional on a recession and as large as 8% conditional on an expansion. For the two-year cumulative multiplier the biases are more modest. Given a 10 standard deviation shock, the asymptotic bias of the state-dependent LP estimator of the four-year cumulative fiscal multiplier reaches close to 40 percent, given a recession. This evidence suggests that the asymptotic bias of the LP estimator can be large enough in realistic settings to be a concern for applied work.[13] We also examined analogous results for $\delta \in \{-1, -5, -10\}$, which are reported in Appendix D, and found asymptotic biases in the cumulative fiscal multiplier that were larger than those for the corresponding positive δ conditional on expansions, and somewhat smaller, but still substantial, conditional on recessions.

[13] One might have expected that the bias in the numerator and denominator of the multiplier would offset, leaving the multiplier largely unaffected. Our simulations show this need not be the case in general. The asymptotic biases in the responses of GDP and government spending have the same sign at some horizons, but opposite signs at other horizons. Moreover, even when the biases in the *LP* estimator of the government spending and GDP responses are of the same sign, their magnitude differs and hence the bias in the cumulative responses typically does not cancel.

It may be tempting to argue that our evidence that the asymptotic bias is at most 4 percent for $\delta = 1$ suggests that as long as δ is not too large, the state-dependent LP estimator will get the multiplier right at least approximately. That appears true in this example, but we cannot rule out that for other specifications of the data generating process this bias could be larger. As we discuss in the next section, a better option would be to replace the state-dependent LP estimator by an alternative estimator that remains asymptotically valid in this setting.

6 An alternative to state-dependent local projections

As we demonstrated, state-dependent local projections generally do not recover the conditional average response function of y_{t+h} to a shock in ε_{1t} of fixed size δ, yet macroeconomists often care about the impact of non-negligible shocks. This poses a quandary for applied researchers who – for one reason or another – are not willing to estimate a fully specified state-dependent SVAR model. In this section, we propose an alternative approach that can be used when the conditional average response function is the impulse response function of interest. While fully developing the theoretical foundations of this estimator (or examining its finite-sample accuracy) is beyond the scope of this paper, we outline the central idea underlying this proposal. Let $g_h(e, s) \equiv E(y_{t+h}|\varepsilon_{1t} = e, S_{t-1} = s)$, for any fixed values e and s in the support of ε_{1t} and S_{t-1}, respectively. Then, under our model assumptions (i.e., model (3) and (4) and Assumptions 1 and 2), we can write $CAR_h(\delta, s)$ as $E(g_h(\varepsilon_{1t} + \delta, s) - g_h(\varepsilon_{1t}, s))$. To see this, note that by the law of iterated expectations,

$$
\begin{aligned}
& E\left[y_{t+h}(e+\delta) - y_{t+h}(e) \,|S_{t-1} = s\right] \\
= \;& E\left[y_{t+h}(e+\delta) \,|\varepsilon_{1t} = e+\delta, S_{t-1} = s\right] - E\left[y_{t+h}(e) \,|\varepsilon_{1t} = e, S_{t-1} = s\right] \\
= \;& E\left[y_{t+h}|\varepsilon_{1t} = e+\delta, S_{t-1} = s\right] - E\left[y_{t+h}|\varepsilon_{1t} = e, S_{t-1} = s\right] \\
\equiv \;& g_h(e+\delta, s) - g_h(e, s),
\end{aligned}
$$

where the first equality follows by the independence between the potential outcomes $y_{t+h}(e)$ and ε_{1t} (see Lemma A.1 in online Appendix A), and the second equality follows because $y_{t+h}(e) =$

y_{t+h} when $\varepsilon_{1t} = e$ and $y_{t+h}(e+\delta) = y_{t+h}$ when $\varepsilon_{1t} = e + \delta$. It follows that

$$\begin{aligned} CAR_h(\delta, s) &\equiv E\left(y_{t+h}(\varepsilon_{1t}+\delta) - y_{t+h}(\varepsilon_{1t}) | S_{t-1} = s\right) \\ &= E\left(g_h(\varepsilon_{1t}+\delta, s) - g_h(\varepsilon_{1t}, s) | S_{t-1} = s\right) \\ &= E\left(g_h(\varepsilon_{1t}+\delta, s) - g_h(\varepsilon_{1t}, s)\right), \end{aligned}$$

where the last equality follows because ε_{1t} is independent of $S_{t-1} = \eta(w_r : r \leq t-1)$ under Assumptions 1 and 2.

This result suggests the following approach to estimating $CAR_h(\delta, s)$. First, we estimate $g_h(e, s) = E(y_{t+h}|\varepsilon_{1t} = e, S_{t-1} = s)$ consistently using the observed sample $\{y_{t+h}, \varepsilon_{1t}, S_{t-1}\}$. When $x_t = \varepsilon_{1t}$, as in the narrative approach to identification, $g_h(e, s)$ is identified from data for these three variables. Since this function is generally a complicated nonlinear function of e and s when S_t is endogenous, we can use a nonparametric regression of y_{t+h} on ε_{1t} and S_{t-1} in this step. Letting $\hat{g}_h(e, s)$ denote this estimator, we then average the difference $\hat{g}_h(\varepsilon_{1t}+\delta, s) - \hat{g}_h(\varepsilon_{1t}, s)$ over the realizations of ε_{1t} in the sample.

Algorithm 6.1 (Nonparametric CAR) *Given a sample $\{y_t, \varepsilon_{1t}, q_t, : t = 1, \ldots, T\}$, proceed in two steps:*

1. *Obtain the nonparametric estimator $\hat{g}_h(e, s) \equiv \widehat{E}(y_{t+h}|\varepsilon_{1t} = e, S_{t-1} = s)$ of $g_h(e, s)$.*

2. *Estimate $CAR_h(\delta, s)$ as*

$$\widehat{CAR}_h(\delta, s) = \frac{1}{T}\sum_{t=1}^{T}\left(\hat{g}_h(\varepsilon_{1t}+\delta, s) - \hat{g}_h(\varepsilon_{1t}, s)\right).$$

This proposal provides a constructive alternative to the use of state-dependent LP estimators in applied work when δ is not negligible and the state of the economy is endogenous.

An in-depth evaluation of the asymptotic and small-sample properties of this nonparametric estimator is the subject of ongoing research. To illustrate how the LP and nonparametric estimates differ in an application of general interest, we apply these estimators to the baseline specification of Ramey and Zubairy (2018) where high and low unemployment states are defined depending on whether the unemployment rate is above or below its sample average of 6.5%.

As in all nonparametric estimation, the bandwidth matters when implementing the nonparametric (NP) impulse response estimator. A well known issue with fixed-bandwidth kernel estimators is that they tend to over-smooth the density in areas where there are more observations and under-smooth in the tails. In addition, fixed-bandwidth kernel estimates can be sensitive to the type of kernel and the method used to select the bandwidth. In contrast, the empirical results in this section are obtained using a generalized-nearest-neighbor-bandwidth kernel with the bandwidth selected by least squares cross-validation. The use of this bandwidth selection method was suggested by Li and Racine (2004) and Racine and Li (2004) and is common in the nonparametric estimation literature. This produces results that are robust to alternative cross-validation methods (least squares or Kullback-Leibler) and to the choice of the kernel for the continuous and categorical variables (Gaussian or Epanechnikov and Aitchison-Aitken or Li-Racine, respectively).[14]

The main question examined by Ramey and Zubairy (2018), among others, is whether the multipliers are higher during periods of slack (high unemployment). Table 3 reports the cumulative multipliers calculated using the state-dependent LP, the NP estimates, and their difference. We show the cumulative multipliers at two- and four-year horizons based on equation (13) for the LP estimator and equation (12) for the NP estimator with $\delta = 1$. This value of δ corresponds to the magnitude of the shock employed in the literature. As illustrated in the first and fourth columns of Table 3, the two- and four-year LP multipliers are very similar across states, whereas the NP multipliers are considerably larger in periods of slack and smaller in periods of no slack. The NP multipliers are up to 33% higher when unemployment is high and as much as 49% lower than the LP multiplier when unemployment is low, consistent with our earlier evidence of large bias in the LP multiplier when δ is much larger than σ_1.

7 Conclusion

When the state of the economy evolves independently of macroeconomic shocks, state-dependent local projections recover in the limit the conditional response function to a shock of size δ,

[14]If we are interested in inference about individual impulse responses, their standard errors can be computed based on the results in Chu et al. (2020). Computing the standard errors for the multipliers is considerably more complicated since these are nonlinear functions of multiple impulse response functions. In particular, standard errors for the multiplier should be robust to serial dependence in the direct regression error for each horizon and to dependence across different horizons and across variables. In our case, inference is further complicated by the fact that we estimate each individual impulse response nonparametrically. Inference for the resulting multiplier would require that we account for the dependence among these individual nonparametric estimates, either by applying the delta method, or by using nonparametric bootstrap methods. Even bootstrapping an individual nonparametric regression for a given a horizon is already challenging (see e.g., Cavaliere, Gonçalves, Nielsen and Zanelli (2023)). It is even less clear how to bootstrap the joint distribution of the impulse responses in this case. We defer this problem to future research.

whether δ is fixed or infinitesimal. More typically, the state of the economy is defined in terms of endogenous model variables such as real output or unemployment. A case in point is recessions as defined by the NBER business cycle committee or the rule of thumb defining recessions as two consecutive quarters of negative output growth. Other typical examples include recessions defined as negative deviations of real output from a moving-average trend or Hodrick-Prescott (HP) filter trend or recessions defined as unemployment rates exceeding some threshold.

When the state of the economy is endogenous with respect to macroeconomic shocks, state-dependent local projections under suitable assumptions will recover the conditional marginal response function with respect to an infinitesimal structural shock. However, they will not recover the average response of the economy to larger structural shocks, conditional on being in a recession or an expansion at the time of this shock. In the latter case, one would expect the state-dependent LP estimator to be accurate only when the shock of interest is much smaller than the sample standard deviation of the exogenous policy shock series. Simulation evidence suggests that the LP estimator of impulse responses and multipliers is strongly biased in many realistic settings.

These problems may in principle be overcome by the use of state-dependent structural VAR models at the cost of added complexity. We noted that our analysis suggests an alternative nonparametric approach that retains the parsimony and ease of estimation of LP estimators, yet preserves the ability to recover the average response of the outcome variable, conditional on the state of the economy at the time of the shock. We showed that this nonparametric estimator of the CAR generates substantially different estimates of the government spending multipliers than that the state-dependent LP estimator in the baseline model of Ramey and Zubairy (2018), consistent with our earlier evidence of asymptotic biases in the LP estimator.

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Table 1: Selected journal articles that employ state-dependent local projections

Fiscal policy	**Fiscal policy (continued)**
Alloza, 2022	Riera-Crichton, Vegh, Vuletin, 2015
Auerbach, Gorodnichenko, 2013	Sheremirov, Spirovska, 2022
Auerbach, Gorodnichenko, 2016	
Berge, De Ridder, Pfajfar, 2021	**Monetary policy**
Bernardini, Peersman, 2018	Albrizio et al., 2020
Bernardini, De Schryder, Peersman, 2020	Albuquerque, 2019
Biolsi, 2017	Alpanda, Granziera, Zubairy, 2021
Boehm, 2020	Auer, Bernardini, Cecioni, 2021
Born, Müller, Pfeifer, 2020	El Herradi, Leroy, 2021
Choi, Shin, Yoo, 2022	Falck, Hoffmann, Hürtgen, 2021
Demirel, 2021	Furceri, Loungani, Zdzienicka, 2018
El-Shagi, von Schweinitz, 2021	Jorda, Schularick, Taylor, 2020
Eminidou, Geiger, Zachariadis, 2023	Santoro et al., 2014
Ghassibe, Zanetti, 2022	Tenreyro, Thwaites, 2016
Jorda, Taylor, 2016	
Klein, 2017	**Uncertainty**
Klein, Polattimur, Winkler, 2022	Cacciatore, Ravenna, 2021
Klein, Winkler, 2021	Tillmann, 2020
Leduc, Wilson, 2012	
Liu, 2022	**Other**
Liu, 2023	De Haan, Wiese, 2022
Miyamoto, Nguyen, Segeyev, 2018	Duval, Furceri, 2018
Owyang, Ramey, Zubairy, 2013	Lastauskas, Stakenas, 2020
Ben Zeev, Zubairy, Ramey, 2023	Loipersberger, Matschke, 2022
Ramey, Zubairy, 2018	Sheng, Sukaj, 2021

Note: The articles listed above appeared in: American Economic Review, American Economic Journal: Macroeconomics, Economic Journal, European Economic Review, IMF Economic Review, International Economic Review, International Journal of Central Banking, Journal of Applied Econometrics, Journal of Economic Dynamics and Control, Journal of International Economics, Journal of International Money and Finance, Journal of Monetary Economics, Journal of Money, Credit and Banking, Journal of Political Economy, Journal of Public Economics, NBER Macroeconomics Annual, Review of Economics and Statistics.

Table 2: Asymptotic bias in the cumulative multiplier

	DGP1		DGP2	
	Expansion	Recession	Expansion	Recession
$\delta = 1$				
Bias in 2 year integral	1.5	1.6	0.3	0.7
Bias in 4 year integral	2.3	3.7	1.1	2.2
$\delta = 5$				
Bias in 2 year integral	2.5	6.9	4.3	3.3
Bias in 4 year integral	6.6	17.9	7.7	11.1
$\delta = 10$				
Bias in 2 year integral	1.7	13.0	5.5	5.7
Bias in 4 year integral	10.2	36.6	12.5	21.1

Table 3: Estimates of cumulative government spending multipliers

	High unemployment			Low unemployment		
	LP	NP	NP − LP	LP	NP	NP − LP
2 year integral	0.60	0.81	0.20	0.59	0.33	-0.37
4 year integral	0.68	0.84	0.15	0.67	0.34	-0.32

Notes: LP corresponds to the local projection estimate, NP is obtained with the nonparametric estimator proposed in Section 6. The data and model specification match those of Ramey and Zubairy (2018).

Figure 1: Asymptotic bias of LP response when $S_t = 1\,(y_t > 0)$, $\delta = 1$ and $\sigma_1 = 0.1$

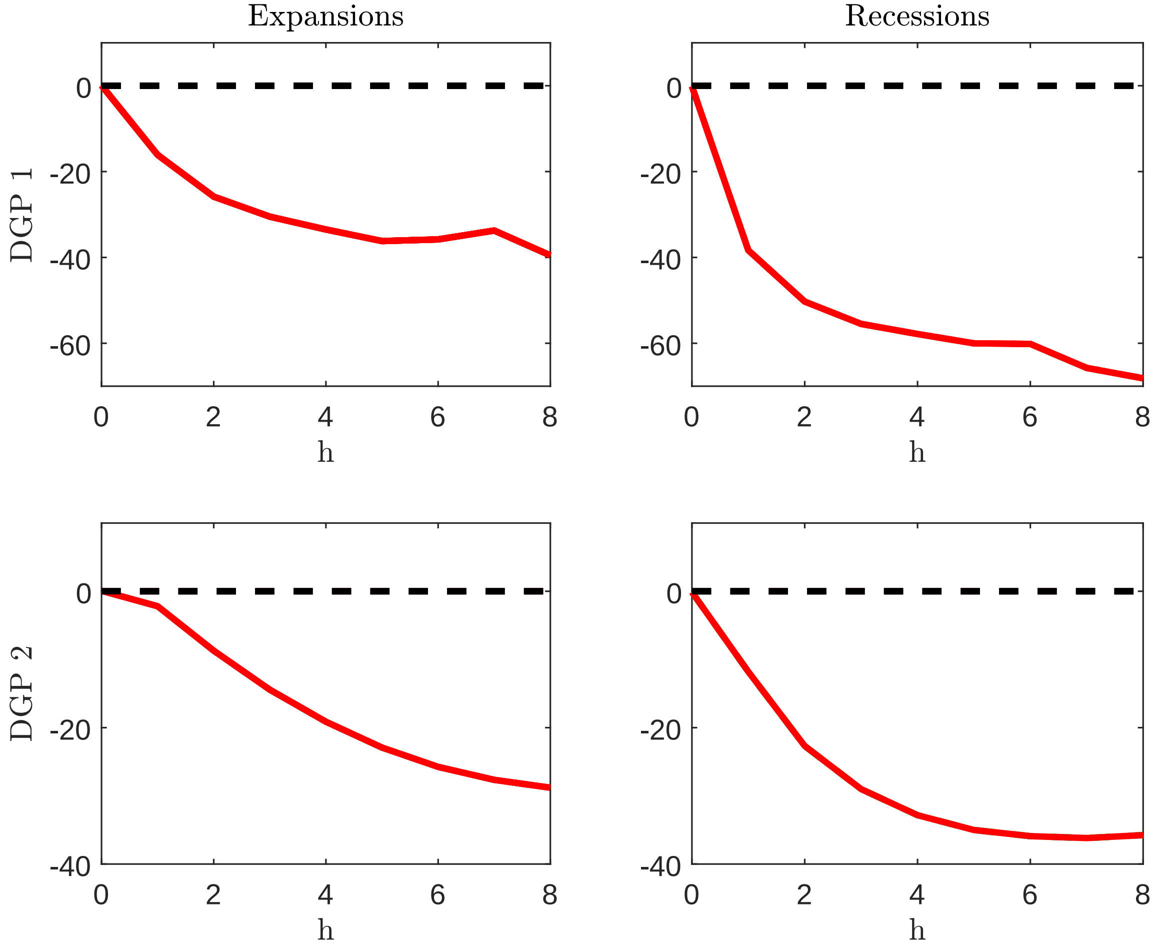

Figure 2: Asymptotic bias of LP response when $S_t = 1\,(y_t > 0)$ and $\sigma_1 = 1$

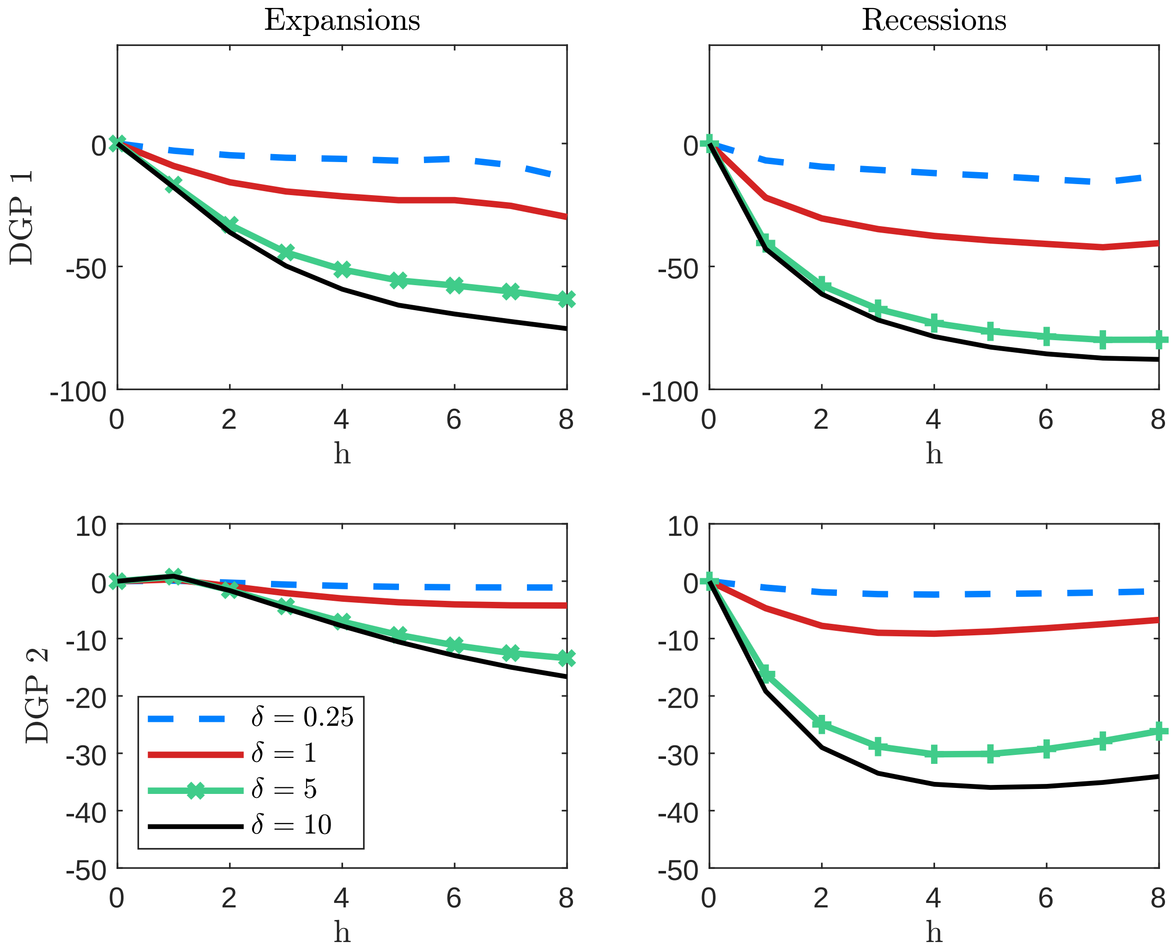

Figure 3: LP response and decomposition of CAR when $S_t = 1\,(y_t > 0)$ and $\delta = 5$

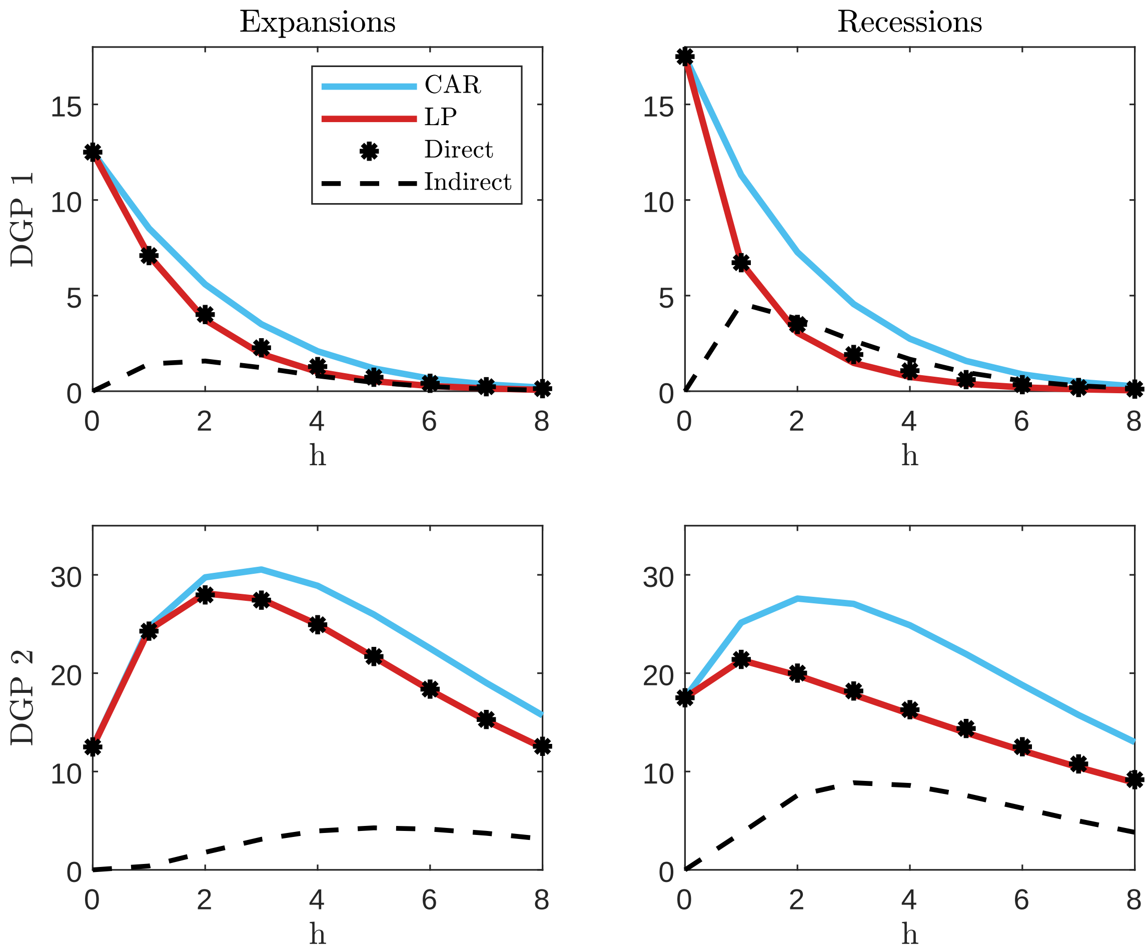

Figure 4: Cumulative spending multiplier when $S_t = 1\,(y_t > 1)$

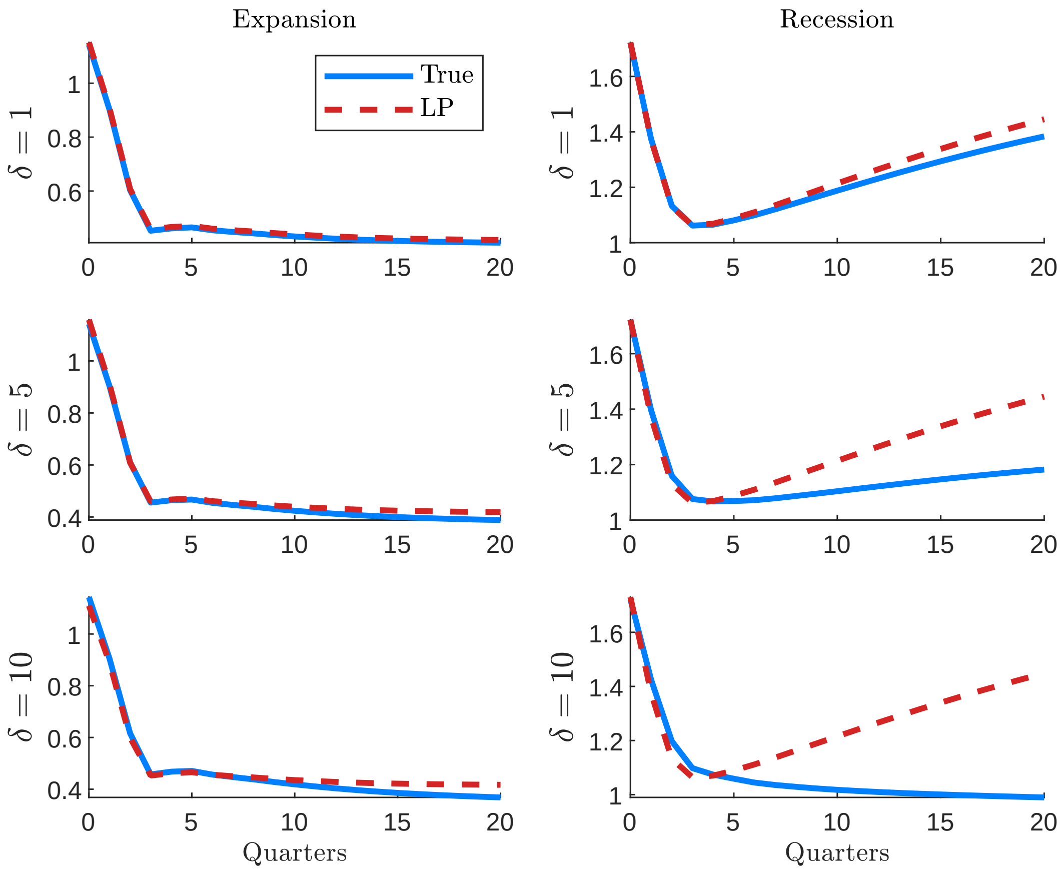

Figure 5: Cumulative spending multiplier when $S_t = 1\,(y_t > MA(12))$